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ABSTRACT

This document focuses on one child's skip counting methods. The pupil, a second grade student at Steuben School, in Kankakee, Illinois, was interviewed as she made several attempts at counting twenty-five poker chips on a circular piece of paper. The interview was part of a larger study of "Children's Conceptions of Number and Numeral," funded by the National Institute of Education (NIE) and directed by Jack Easley and Harold Lerch. The study indicated that the five principles of counting as proposed by Rochel Gelman and C. R. Gallistel are useful, but do not go far enough. Three additional principles are proposed and described. (MP)

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COMPLEXITIES OF COUNTING

by
Bernadine Evans Stake

A search for understanding children's conceptual frameworks in mathematics, of which this study is a part, began in the fall of '78 at Steuben School, Kankakee, Illinois. Children in two classrooms (Kindergarten and second grade) left their classes 1 by 1, sometimes in pairs for videotaped interview sessions. The investigation was a part of a larger study of "Children's Conceptions of Number and Numeral," funded by NIE and directed by Jack Easley, and Harold Lerch.

Interviews with five primary school children were selected from over 200 videotaped interviews and the tapes were analyzed to discover children's conceptions of counting. The clinical interviews focused on place value, addition, subtraction, and multiplication. Five counting principles proposed by Gelman and Gallistel (1978) and three additional principles proposed by the author gave focus to children's counting tactics.

Easley's method of using Piaget's notion of schemas was used to represent children's processes of counting. The usual learning theories (e.g., Thorndike, Skinner) do not predict that children's thinking will shift as drastically as observed in the children at Kankakee. Interpretations of schemas helps to explain those drastic shifts that teachers find difficult to understand.

I found that most children's counting is procedure dominated and often has little relationship to a quest for numerosity. Process schemas seemed separate from quantity schemas. Each of them operated at different times to solve counting problems. Children counted but often not to find the answer to "How many?"

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*This paper contains excerpts from the authors dissertation, Clinical Studies of Counting problems with Primary School Children: which contains a more comprehensive study of children's counting problems completed in 1980 at the University of Illinois. It is available from Dissertation Abstracts.

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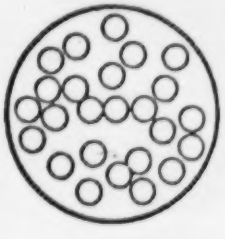
The following account focuses on one child's skip counting.

Katia, Grade 2

Steuben School, Kankakee, Illinois, 10/24/78

At the onset of the interview Katia faced 25 poker chips on a circular piece of paper. The chips were placed randomly. When coaxed Katia said she would call the chips "cookies." She counted the cookies by pushing them aside one by one.

25 chips (cookies) on a circular paper. Katia pushes chips aside as she counts.



K: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25.

I: 25. Do you think that's right?

K: (Nods in agreement)

I: Are you pretty sure?

K: (Nods in agreement and smiles)

Katia counted by 1s in a direct one-to-one correspondence, pushing aside cookies one by one, tagging them once and only once. It seemed that Katia had determined the numerosity. I repeated the last number from Katia's counting; Katia agreed. It seemed that she understood the cardinality of the set at this point. As the interview continued Katia made responses inconsistent with that assumption.

Katia pushes one chip at a time as she "counts by 5s the same 25 chips in a one-to-one correspondence (one numeral to one object).

I: Could you count them in some other way?

K: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 75, 80, 85, 90, (pause) 100, 101, 102, 103, 106, 107, 108.

Katia looks at (I) and repeats the last numeral.

K: one hundred eight

In this interview with Katia I inferred that the counting schema: one-to-one correspondence (one object to one numeral) was in operation much of the time. That schema, however, had subschemas based on different lists of number names. (See Figure 1.) Katia counted in a direct one-to-one correspondence but changed the list of number names when she counted by 5s. Up to 100 her counting was one-to-one correspondence by 5s.

Such schemas and subschemas are seen to operate like a "room full of experts." When problems are presented, one may step forward and respond to the problem. Certain schemas (experts) are operative at any one time; when a question is posed to a child, an expert takes over. The child's response to the problem depends on which schema is operating in the child's mind (which expert steps forward).

Katia appeared confident with the counting procedures, which may have accounted for her confidence in the results. It might be said that she was "procedure confident." The procedure perhaps had become so important that even though the use of the procedure produced a different result, each result was perhaps primarily the signification that the procedure had been completed. Katia's schemas appeared to be consistent with the principles of Gelman and Gallistel. But the use of those principles was not sufficient to produce the correct answer to the question, "How many?"

I thought Katia should have recognized the jarring discrepancy.

I: When you counted by 5s... (Pause). Show me again how you count by 5s.

Katia pushes aside one "cookie" for each numeral she repeats and emphasizes 105.

K: 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80, 85, 90, 95, 100, 101, 102, 103, 105---<u>one hundred five</u>.

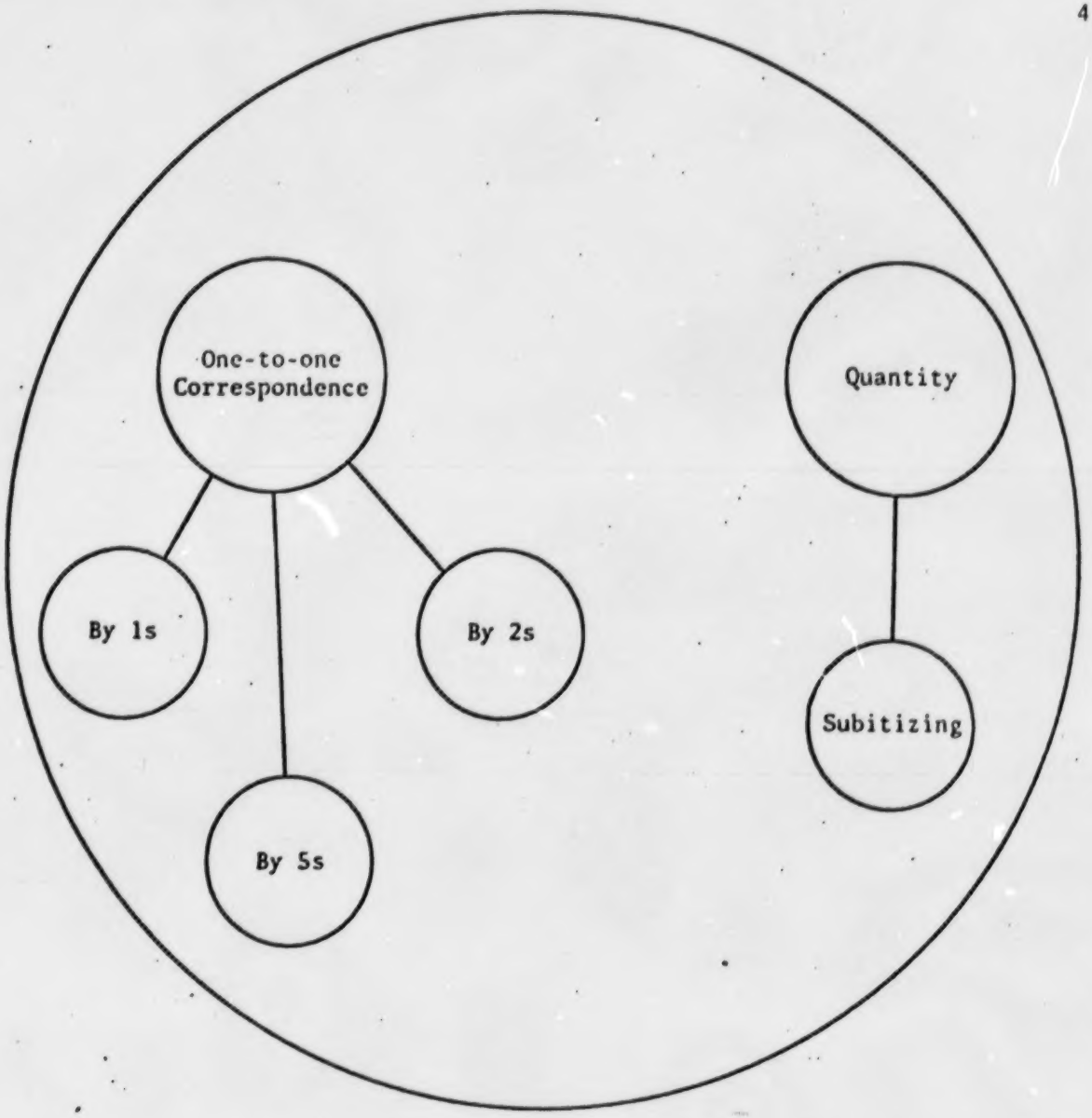


Figure 1. Katia's Counting Schemas

I: (Pause) You got 105? Now these are cookies and you have them for Halloween. Right? They'll be Halloween cookies. You have a hundred friends coming over. They each want a cookie. Do you have enough for all of them? For each one to have a cookie?

K: (Nods "yes")

I: You do? Think so?

K: (Nods in agreement)

(I) moves one chip away from the others, then points to it.

I: Let's see now. When you counted like this and you said, "5." What did it mean? That you have 5 cookies right there?

K: No.

Katia also counted the 25 chips by 2s in a direct one-to-one correspondence, i.e., one object for one number name. She hesitated at first, placing her hand on two cookies, as if to follow a proper two cookies to one number-name correspondence. Then she reverted to one-to-one correspondence. A quantity schema may have taken over momentarily, but it was not maintained.

I moved the chips into five clusters of five and asked Katia if there were really 105 cookies. Katia seemed to understand there was a problem.

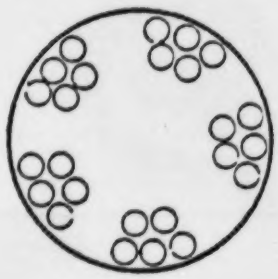
I: When you count by 5s..... Let's think about it. If you count by 5s, what do you do?

K: (No response)

(I) removes the chips, then puts 5 in a cluster and counts.

I: What if I said, "the way I count by 5s is I take 1, 2, 3, 4, 5?"

(I) positions chips as shown below:



(K) watches

(I) points to one cluster of 5 chips

I: OK, If I were going to count by 5s I might count: How many here?

(K) counted

(I) points to a second cluster of 5 chips

K: Five

I: Here:

K: Ten.

(I) points to the third cluster.

I: Here?

(K) appears to be counting by 5s to herself

K: 35!

I: Now, tell me how you got 35.

(K) puts her hand on one cluster of 5, then on another cluster

K: Count by 5s. . . . ten.

I: Um-hum. Count out loud. Can you think out loud for me, so I can figure it out? See, I'm trying to figure out what children think.

(K) points to two set of five as she counts.

K: 5, 10.

Then she points to one cookie at a time in a third set. She continues in this manner until all but one is counted. The twenty-fifth chip is not counted.

15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, 70, 75, 80.

K: 80?

I: 80? What do you think? Is that right?

K: (Nods head yes)

We might say the quantity "expert" was at work as Katia counted the first "five, ten," but that the one-to-one expert was separate from "counting by 5s." The one-to-one correspondence "expert" took over as Katia counted the rest of the cookies. For fewer objects Katia seemed to have the sense of quantity. The larger number of objects seemed to have little quantity meaning for her.

Katia perhaps began to find two schemas--the one-to-one correspondence schema and the quantity schema--in conflict. She said, "All of them together is 25," which made me think Katia then understood something about the quantity of the collection of chips (or at least recognized that the counting-by-1s result was better than the counting-by-5s result). Covering two clusters of five I asked Katia to count the 3 sets of 5 by 1s.

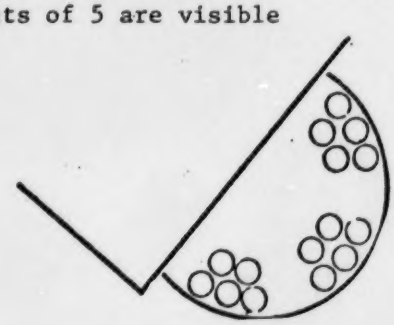
Katia points to each "cookie" and counts to 15...then looks at (I).

K: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15,

I: How many?

K: 15

(I) covers 2 sets of 5 so three sets of 5 are visible



I: What do you think there are...Lets count by 5s again. Start there.

K: 5, 10, 15

Katia counted the first three sets as if sensitive to quantity. Then when four sets were in view she counted the next set of five objects, 20, 25, 30, 35, 40, 45--adding one more numeral than object.

SUMMARY

From their developmental view of children counting, Rochel Gelman and C.R. Gallistel proposed five principles of counting. Up to a point they showed a remarkable agreement with our children's behavior in counting. The principles are useful but do not go far enough.

The first principle, requiring one-to-one correspondence between objects and number tags, clearly is the goal, for most primary children with most counting tasks. The second principle, maintaining a stable order of number tags, is one children work hard at. The third principle called by Gelman and Gallistel the "cardinal principle," operated very strongly in most children as they almost without exception chose the final tag as the answer to the question, "How many?" The fourth principle, ability to count non-physical entities, was not studied. The fifth principle, order irrelevance, was not a problem. Our children were able to proceed in any order through objects as they counted.

The cardinality principle would seem to lie at the heart of understanding quantity. In these interviews, it appeared that this principle was followed by almost all the children. Their final tag was repeated with emphasis; under questioning they proclaimed the number stated was the correct answer to the question, "How many?" Yet, within a minute they frequently declared that a different amount was correct. They sometimes arrived at different results, because they did not make the appropriate adjustment in the correspondence principle when skip counting.

Gelman and Gallistel might have anticipated such problems of understanding quantity because they suggested that young children's cognitive principles are ready to assimilate whatever list becomes available. As soon as two or more sequences of number tags are learned this problem arises. Steuben School children used many different sequences; for example, they counted backward; others (including Katia) skip counted.

Gelman and Gallistel would not be surprised that these children made mistakes, nor that they tolerated contradictions distressing to a teacher; but readers of their book probably would be surprised that their principles had so little to do with understanding the quantity of a set of objects. Part of the problem is that children who are sophisticated in counting take steps to protect their counting from common errors and to check the amount by other means--steps that unsophisticated counters do not take. These steps, which may be involved in developing a useful understanding of number as well as confidence in tackling difficult problems, were not adequately indicated in the principles of Gelman and Gallistel.

In addition to the principles proposed by Gelman and Gallistel there seemed to be a need for additional principles. The three I suggest are: (6) collections need to remain fixed in size, (7) the step size of the list of number names must equal the quantity of each subset of objects tagged (in skip counting this was often violated) and (8) the final tag must be compatible with other evidence of cardinality. Counting in complex tasks make principles (1) and (3) difficult to follow so latest results should be compared with other data.

The first of these suggested principles (6), although very obvious was important in the interviews at Steuben School because additional objects (often lying at one's finger tips) occassionally slipped into the collection being counted. Violations of this principle are consistent with children's acceptance of inconsistent results.

The second additional principle (7)--matching step size--was violated frequently in the interviews analyzed in which children skip counted with alternative stable orders of number names and tagged <u>each</u> object with one of the number names.

The last principle (8) is one which we found frequently ignored, but if children had sufficient time and persisting questioners, as they had here, they eventually gave evidence that they had compared results and used several sources of information to estimate the accuracy of an answer. They were not always comfortable doing this and were often inept in discussing their process of reasoning. It may be that they had too

little sense of the constancy of a task or the conservation of quantity. As has been stated often, they appeared to announce the cardinality of the set more to indicate the completion of the process than to declare the quantity.